EE2030010: Linear Algebra **Spring 2018**
Department of Electrical Engineering
National Tsing Hua University

Homework #3
Coverage: chapter 1-7
Due date: 15 June 2018

Instructor: Chong-Yung Chi *TAs: Amin Jalili & Yao-Rong Syu*

Notice:
1. *Please hand in the hardcopy of your answer sheets to the TAs by yourself before 23:59 of the due date. No late homework will be accepted.*
3. *Ansewrs to the problem set should be written on the A4 papers.*
4. *Write your name, student ID, email and department on the begining of your ansewr sheets.*
5. *Please justify your answers with clear, logical and solid reasoning or proofs.*
6. *Please do the homework independently by yourself. However, you may discuss with someone else but copyied homework is not allowed. This will show your respect toward the academic integrity.*
7. *Your legible handwriting is fine. However, you are very welcome to use text formatting packages for writing your answers.*

Problem 1. (10 points) Let $\mathbf{A} = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$. Find the QR decomposition such that $\mathbf{A} = \mathbf{QR}$ where $\mathbf{Q}$ and $\mathbf{R}$ are the orthogonal and uppertriangular matrices, respectively.

Problem 2. (10 points) Let $\mathbf{A} = \begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix}$. Find determinant of $\mathbf{A}$ based on x, y, z using row operations.

Problem 3. (10 points) Let $\mathbf{A}$ be a projection matrix. Show that $N(\mathbf{A}) = R(\mathbf{I} - \mathbf{A})$, where $\mathbf{I}$ is the identity matrix, $R(\cdot)$ and $N(\cdot)$ are the range space (i.e., column space) and null space of corresponding matrix, respectively.

Problem 4. (10 points) Let $\mathbf{A} = \begin{bmatrix} 0 & 9 \\ -4 & 0 \end{bmatrix}$.

(a) Find the eigenpairs of $\mathbf{A}$.

(b) Let $\mathbf{A} = \mathbf{U\Sigma V}^T$ be the singular value decomposition of $\mathbf{A}$. Find $\mathbf{U}$, $\mathbf{\Sigma}$ and $\mathbf{V}$.

Problem 5. (10 points) Let $d = r + st^2$, where $r, s \in \mathbb{R}$ are constants, provides the best least squares fit to the points $(t_1, d_1) = (1, 1)$, $(t_2, d_2) = (2, 4)$ and $(t_3, d_3) = (4, 8)$.

(a) Find r and s of the best curve $d = r + st^2$ with the best least squares fit to these three points.

(b) Find values of d_1, d_{12} and d_3 at times $t_1 = 1$, $t_2 = 2$ and $t_3 = 4$ such that the best fitting cure is $d = 0$.

Problem 6. (10 points) Define the matrix $\mathbf{A}$ as

$$\begin{array}{rc}
n \times 2n \left\{ \begin{array}{c} \vphantom{1} \\ \vphantom{1} \\ \vphantom{\vdots} \\ \vphantom{1} \end{array} \right. & \\[-4.5em]
& \begin{bmatrix}
1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \\
2 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\
0 & 2 & \cdots & 0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 2 & 0 & 0 & \cdots & 1
\end{bmatrix} \\
& \underbrace{\hphantom{1\quad 0\quad \cdots\quad 0}}_{2n \times n} \hphantom{\quad 0\quad 0\quad \cdots\quad 0}
\end{array}$$

be a $2n \times 2n$ matrix. Find $\mathbf{A}^{\ell}$ where ℓ is a positive integer number.

Problem 7. (10 points) Let $\mathbf{A}$ be an $n \times n$ matrix.

(a) Considering $(\lambda, \mathbf{v})$ as an eigenpair of $\mathbf{A}$, what can be the associated eigenpair of $\mathbf{A}^k$ where k is positive integer?

(b) Suppose $\mathbf{A}$ is a nilpotent matrix (i.e., $\mathbf{A}^k = 0$ for $k \geq 2$). What is the eigenvalue of $\mathbf{A}$ in this case?

Problem 8. (15 points) Let $\mathbf{A}$ be an invertible matrix.

(a) Show that for any eigenvalue λ of $\mathbf{A}$, λ^{-1} is an eigenvalue of $\mathbf{A}^{-1}$.

(b) Show that the eigenspace of $\mathbf{A}$ corresponding to λ is the same as the eigenspace of $\mathbf{A}^{-1}$ corresponding to λ^{-1}.

(c) Show that if $\mathbf{A}$ is diagonalizable, then $\mathbf{A}^{-1}$ is also diagonalizable.

Problem 9. (15 points) Let $\mathbf{A} \in \mathbb{R}^{n \times n}$ be invertible. Prove that there exist unique matrices $\mathbf{Q}$ and $\mathbf{P}$ which are orthogonal and positive-semidefinite matrices, respectively such that $\mathbf{A} = \mathbf{QP}$.